LESSON 61

- **Remaining Fraction**
- **Two-Step Equations**

Power Up

facts

Power Up I

count aloud

Count by halves from $\frac{1}{2}$ to 6 and back down to $\frac{1}{2}$.

mental math

In problems **a–c,** multiply a number by 10.

a. **Number Sense:** 12×10

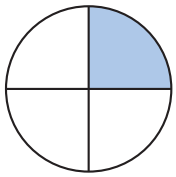
b. **Number Sense:** 120×10

c. **Number Sense:** 10×10

d. **Money:** Jill paid for a pencil that cost 36¢ with a $1 bill. How much change should she receive?

e. **Money:** One container of motor oil costs $3.75. How much do 2 containers cost?

f. **Percent:** The whole circle has been divided into quarters. What percent of the circle is shaded? What percent is not shaded?



g. **Estimation:** Phil plans to buy lasagna for $5.29 and a drink for $1.79. Round each price to the nearest 25 cents and then add to estimate the total cost.

h. **Calculation:** $48 + 250 + 6 + 6$

problem solving

Choose an appropriate problem-solving strategy to solve this problem. Garcia is packing his clothes for summer camp. He wants to take three pairs of shorts. He has four different pairs of shorts from which to choose—tan, blue, white, and black. What are the different combinations of three pairs of shorts that Garcia can pack?

New Concepts

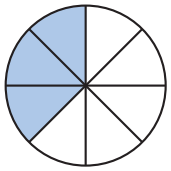
Remaining Fraction

The whole circle in Example 1 below has a shaded portion and an unshaded portion. If we know the size of one portion of a whole, then we can figure out the size of the other portion.

Example 1

a. What fraction of the circle is shaded?

b. What fraction of the circle is not shaded?



We see that the whole circle has been divided into eight equal parts. Three of the parts are shaded, so five of the parts are not shaded.

a. The fraction that is shaded is $\frac{3}{8}$.

b. The fraction that is not shaded is $\frac{5}{8}$.

Represent Compare the shaded part to the part not shaded using $>$, $<$, or $=$.

Example 2

The pizza was cut into eight equal slices. After Willis, Hunter, and Suelita each took a slice, what fraction of the pizza was left?

The whole pizza was cut into eight equal parts. Since three of the eight parts were taken, five of the eight parts remained. The fraction that was left was $\frac{5}{8}$.

Example 3

Two fifths of the crowd cheered. What fraction of the crowd did not cheer?

We think of the crowd as though it were divided into five equal parts. We are told that two of the five parts cheered. So there were three parts that did not cheer. The fraction of the crowd that did not cheer was $\frac{3}{5}$.

Two-Step Equations

The equation below means, "2 times what number equals 7 plus 5?"

$$2n = 7 + 5$$

It takes two steps to solve this equation. The first step is to add 7 and 5 ($7 + 5 = 12$), which gives us this equation:

$$2n = 12$$

The second step is to find n. Since $2 \times 6 = 12$, we know that n is 6.

$$n = 6$$

Verify How can we check the answer?

Example 4

Find m in the following equation: $3m = 4 \cdot 6$

A dot is sometimes used between two numbers to indicate multiplication. So $4 \cdot 6$ means "4 times 6." The product of 4 and 6 is 24.

Reading Math

We read this equation as "3 times what number equals 4 times 6?"

$$3m = 4 \cdot 6$$

$$3m = 24$$

Now we find m. Three times 8 equals 24, so m equals 8.

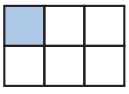
$$3m = 24$$

$$m = \mathbf{8}$$

Verify How can we check the answer?

Lesson Practice

a. What fraction of this rectangle is not shaded?



b. Three fifths of the race was over. What fraction of the race was left?

Find each missing number:

c. $2n = 2 + 8$

d. $2 + n = 2 \cdot 8$

Written Practice

Distributed and Integrated

*** 1.** (21) **Explain** The diameter of Filomena's bicycle tire is 24 inches. What is the radius of the tire? Explain how you know.

2. (49) There are five apple slices in each school lunch. If 35 students buy a school lunch, how many apple slices are there? Write an equation for this problem.

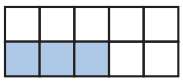
3. (36, Inv. 4) **a.** Two nickels are what fraction of a dollar?

b. Two nickels are what decimal part of a dollar?

***4.** (60) The Gilbreth family drank 39 cups of milk in 3 days. That averages to how many cups of milk each day?

***5.** (1, 17) Maya drove 28 miles to Ariana's house. That afternoon the two friends drove 3 miles to a restaurant and then drove back to Ariana's house. That evening Maya drove 28 miles to return home. Altogether, how many miles did Maya travel that day?

***6.** (61) What fraction of this rectangle is *not* shaded?



***7.** (55) **Multiple Choice** Which of these numbers is *not* a factor of 10?

A 2 **B** 5 **C** 10 **D** 20

***8.** (61) **Verify** The loaf of bread was sliced into 6 equal pieces. After 1 piece was taken, what fraction of the loaf was left?

***9.** (56) **Represent** Compare these fractions. Draw and shade two congruent circles to show the comparison.

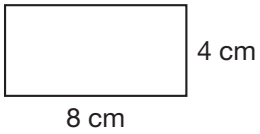
$$\frac{2}{3} \bigcirc \frac{3}{4}$$

***10.** (54, 59) **Estimate** Find the sum of 5070 and 3840 by rounding each number to the nearest thousand before adding.

11. (Inv. 5) If 60% of the answers were true, then were there more true answers or more false answers?

12. (Inv. 2, Inv. 3) **a.** What is the perimeter of this rectangle?

b. What is the area of this rectangle?



13. (43, 51)
$$\begin{array}{r} \$62.59 \\ +\ \$17.47 \\ \hline \end{array}$$

***14.** (61) $5n = 12 + 18$

***15.** (45, 58) $1000 - (110 \times 9)$

16. (50) $3.675 - 1.76$

***17.** (58)
$$\begin{array}{r} \$6.70 \\ \times \quad 4 \\ \hline \end{array}$$

***18.** (58)
$$\begin{array}{r} 703 \\ \times \quad 6 \\ \hline \end{array}$$

***19.** (58)
$$\begin{array}{r} \$346 \\ \times \quad 9 \\ \hline \end{array}$$

***20.** (53) $5\overline{)39}$

***21.** (53) $7\overline{)39}$

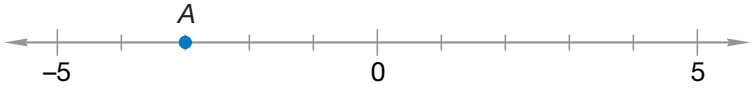
22. (53) $4\overline{)39}$

23. (53) $16 \div 3$

24. (53) $26 \div 6$

25. (Inv. 3, 47) $36 \div \sqrt{36}$

***26.** (Inv. 1) **Represent** Point *A* represents what number on this number line?



27. (33) Compare:

a. 745 ◯ 754

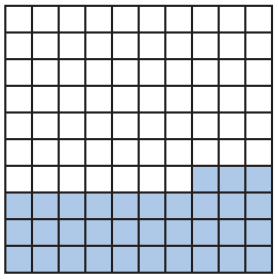
b. 132 ◯ 99

28. (Inv. 4, Inv. 5)

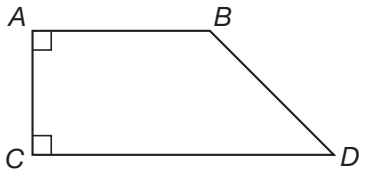
a. What fraction of the large square is not shaded?

b. The unshaded part of the large square represents what decimal number?

c. What percent of the large square is not shaded?



***29.** (52) **Classify** Name the parallel and perpendicular segments in this figure. Describe the angles as acute, obtuse, or right.



***30.** (23, 45) In 1847 the first adhesive postage stamps were sold in the United States. In 1873 the first postcards were issued. What is the elapsed time in years between those two events?

LESSON 62

- **Multiplying Three or More Factors**
- **Exponents**

Power Up

facts

Power Up I

count aloud

Count by fourths from $5\frac{1}{4}$ to 10.

mental math

a. **Number Sense:** 14×10

b. **Money:** Sean bought a ream of paper for $6.47 and a box of staples for $1.85. What was the total cost?

c. **Percent:** Compare: 25% ◯ $\frac{1}{2}$

d. **Geometry:** What is the perimeter of a square that is 6 inches on each side?

e. **Time:** Crystal phoned her friend at 4:05 p.m. They talked for 22 minutes. What time did Crystal's phone call end?

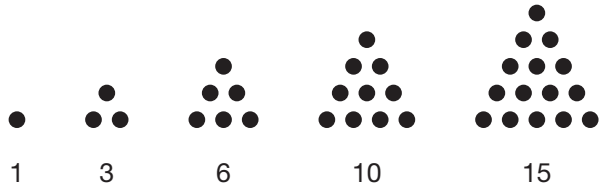
f. **Measurement:** Ray cut a 1-foot length of string from a larger piece that was 22 inches long. How many inches of string remained?

g. **Estimation:** Washington School has 258 students. Lincoln School has 241 students. Round each number to the nearest ten and then add to estimate the total number of students.

h. **Calculation:** $400 + 37 + 210 - 17$

problem solving

Choose an appropriate problem-solving strategy to solve this problem. The following page shows a sequence of triangular numbers. The third term in the sequence, 6, is the number of dots in a triangular arrangement of dots with three rows. Notice that in this sequence the count from one number to the next increases. Find the number of dots in a triangular arrangement with 8 rows. Explain how you arrived at your answer and how you can verify your answer.



New Concepts

Multiplying Three or More Factors

To find the product of three numbers, we first multiply two of the numbers. Then we multiply the answer we get by the third number. To multiply four numbers, we must multiply once more. In any multiplication we continue the process until no factors remain.

Example 1

Multiply: $3 \times 4 \times 5$

First we multiply two of the numbers to get a product. Then we multiply that product by the third number. If we multiply 3 by 4 first, we get 12. Then we multiply 12 by 5 and get 60.

Step 1	Step 2
3	12
$\times$ 4	$\times$ 5
12	**60**

It does not matter which two numbers we multiply first. If we multiply 5 by 4 first, we get 20. Then we multiply 20 by 3 and again get 60.

Step 1	Step 2	
5	20	
$\times$ 4	$\times$ 3	
20	**60**	← same answer

The order of the multiplications does not matter because of the Associative Property of Multiplication, which we studied in Lesson 45.

Example 2

Multiply: $4 \times 5 \times 10 \times 10$

We may perform this multiplication mentally. If we first multiply 4 by 5, we get 20. Then we multiply 20 by 10 to get 200. Finally, we multiply 200 by 10 and find that the product is **2000.**

Exponents

An **exponent** is a number that shows how many times another number (the **base**) is to be used as a factor. An exponent is written above and to the right of the base.

base → 5^2 ← exponent

5^2 means 5×5.

5^2 equals 25.

If the exponent is 2, we say "squared" for the exponent. So 5^2 is read as "five squared." If the exponent is 3, we say "cubed" for the exponent. So 2^3 is read as "two cubed."

Example 3

Simplify: $5^2 + 2^3$

We will add five squared and two cubed. We find the values of 5^2 and 2^3 before adding.

5^2 means 5×5, which is 25.

2^3 means $2 \times 2 \times 2$, which is 8.

Now we add 25 and 8.

$$25 + 8 = \mathbf{33}$$

Example 4

Rewrite this expression using exponents:

$$\mathbf{5 \times 5 \times 5}$$

Five is used as a factor three times, so the exponent is 3.

$$\mathbf{5^3}$$

Math Language

An **exponential expression** indicates that the base is to be used as a factor the number of times shown by the exponent.

$4^3 = 4 \times 4 \times 4$

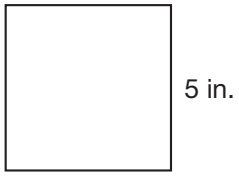
Exponents are sometimes used in formulas. The formula for the area of a square shows that the length of a side (s) is squared.

$$A = s^2$$

Area of a square = (length of side)^2

Example 5

Use the formula for the area of a square to find the area of this square.



The formula for the area of a square is $A = s^2$. The length of each side is 5 in. Replace the "s" in the formula with 5 in.

$$A = (5 \text{ in.})^2$$

Multiplying 5 in. × 5 in., we find the area of the square is 25 sq. in.

We can write the answer as **25 in.^{2}** or **25 sq. in.**

Reading Math

We read s^2 as "s squared" which means $s \times s$.

Lesson Practice

Simplify:

a. $2 \times 3 \times 4$

b. $3 \times 4 \times 10$

c. 8^2

d. 3^3

e. $10^2 - 6^2$

f. $3^2 - 2^3$

g. Rewrite this expression using exponents:

$$4 \times 4 \times 4$$

h. Write a formula for finding the area of a square. Then use the formula to find the area of a square with each side 6 inches long.

Written Practice

Distributed and Integrated

Formulate Write and solve equations for problems **1** and **2.**

(52) ***1.** There were twice as many peacocks as there were hens. If there were 12 peacocks, then how many hens were there?

(43, 59) ***2.** Mae-Ying bought a package of paper priced at $1.98 and 2 pens priced at $0.49 each. The tax on the entire purchase was 18¢. What was the total cost of the items? Explain why your answer is reasonable.

(27) **3.** Raquel's dance class begins at 6 p.m. It takes 20 minutes to drive to dance class. What time should she leave home to be on time for dance class?

(57) ***4.** **Analyze** Glenda drove across the desert at an average speed of 60 miles per hour. At that rate, how far would she drive in 4 hours? Make a table to solve the problem.

(61) ***5.** Two thirds of the race was over. What fraction of the race was left?

(59) ***6.** **Estimate** Otieno bought a notebook for $8.87 and paper for $2.91. Estimate the total by rounding each amount to the nearest dollar, then add.

(61) ***7.** In the equation $9 \times 11 = 100 - y$, the letter y stands for what number?

*8. (56) **Represent** Compare: $\frac{2}{4} \bigcirc \frac{4}{8}$. Draw and shade two congruent circles to show the comparison.

*9. (55) **Multiple Choice** Recall that a prime number has exactly two factors. Which of these numbers has exactly 2 factors?

A 7 **B** 8 **C** 9 **D** 10

10. (54) According to this calendar, July 4, 2014 is what day of the week?

JULY 2014

S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

*11. (47) **Connect** Write four multiplication/division facts using the numbers 6, 3, and 18.

*12. (62) $5 \times 6 \times 7$

*13. (62) 4^3

14. (51) $476{,}385 + 259{,}518$

15. (52) $\$20.00 - \$17.84$

16. (24) $c - 19{,}434 = 45{,}579$

*17. (58) $\$4.17 \times 8$

*18. (58) $\$470 \times 7$

*19. (58) 608×4

20. (53) $4\overline{)29}$

21. (53) $8\overline{)65}$

22. (53) $5\overline{)29}$

23. (53) $65 \div 7$

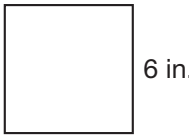
24. (53) $29 \div 5$

25. (53) $65 \div 9$

26. (Inv. 5) If 40% of the students are boys, then what percent of the students are girls?

*27. (Inv. 2, Inv. 3) **a.** What is the perimeter of this square shown at right?

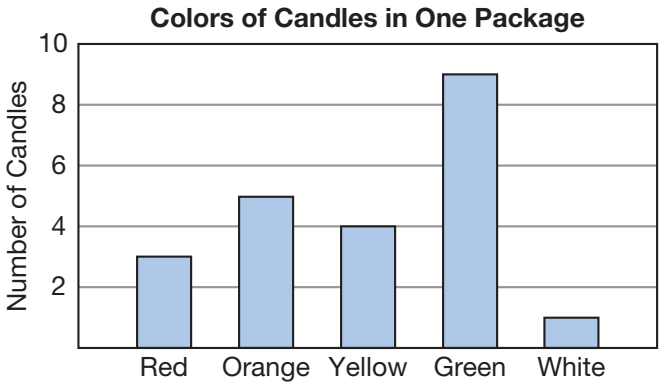
b. Use a formula to find the area of the square.



*28. **Multiple Choice** What type of angle is each angle of a square? (23)

A acute **B** right **C** obtuse **D** straight

*29. This bar graph shows the number of colored candles in a package. Use the bar graph to answer each question. (Inv. 6)



a. How many red candles were there?

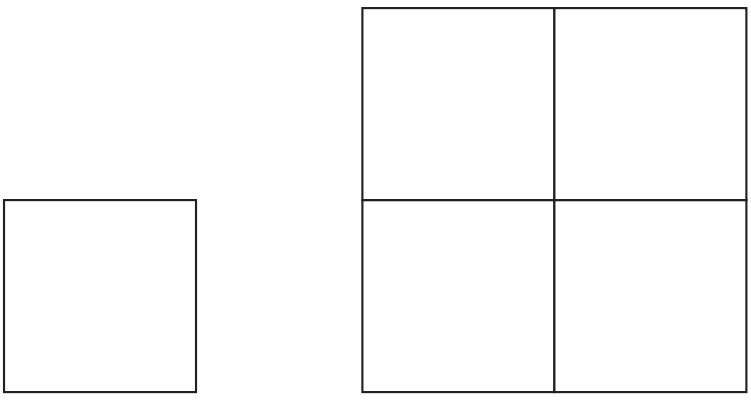
b. There were how many more green candles than orange candles?

*30. **Model** Draw a number line from 1 to 2, and show the locations of $1\frac{1}{2}$, 1.25, and $1\frac{3}{4}$. (37, 50)

Early Finishers

Real-World Connection

A square with 1-inch sides has an area of 1 square inch. A square with 2-inch sides has an area of 4 square inches. Review the squares shown below.



1 × 1 = 1 square inch

2 × 2 = 4 square inches

a. What is the area of a square with sides that are 3 inches long?

b. What is the area of a square with sides that are 4 inches long?

c. What is the area of a square with sides that are 5 inches long?

d. Draw and label each of the squares in parts **a–c.**

LESSON 63

• Polygons

Power Up

facts Power Up J

count aloud Count down by thousands from 20,000 to 1000.

mental math Multiply three numbers in problems **a–c.**

a. **Number Sense:** $6 \times 7 \times 10$

b. **Number Sense:** $5 \times 8 \times 10$

c. **Number Sense:** $12 \times 10 \times 10$

d. **Money:** $\$7.59 + \$0.95$

e. **Money:** Sydney had $5.00. Then she spent $3.25 on photocopies. How much money does she have left?

f. **Geometry:** Compare: $4\frac{1}{2}$ in. ◯ radius of a circle with a 10 in. diameter

g. **Estimation:** Henry estimated that his full drinking glass contained 400 mL of water. Is this a reasonable estimate?

h. **Calculation:** $470 - 30 + 62 + 29$

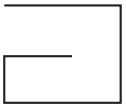
problem solving Choose an appropriate problem-solving strategy to solve this problem. Fifty percent of the students in Gabriel's class are girls. Do we know how many students are in this class? Do we know whether there are more boys or more girls in the class? Do we know whether the number of students in the class is even or odd?

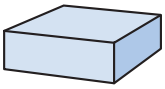
New Concept

Polygons are closed, flat shapes formed by line segments.

Example 1

Which of these shapes is a polygon?

A 

B 

C 

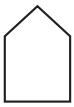
D 

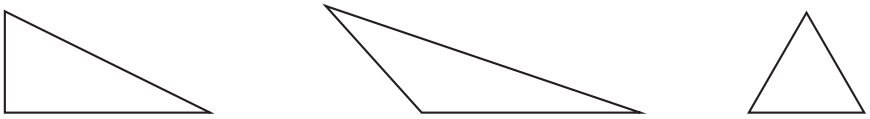
Figure A is not a polygon because it is not closed. Figure B is not a polygon because it is not flat. Figure C is not a polygon because not all of its sides are straight. **Figure D** is a polygon. It is closed and flat, and each of its sides is a line segment.

Thinking Skill

Verify

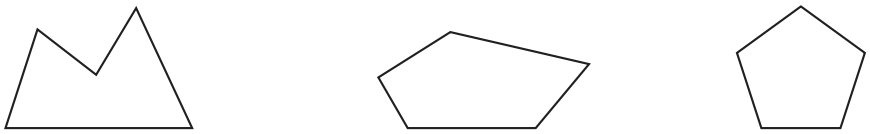
What is a regular rectangle called?

Polygons are named according to the number of sides they have. The lengths of the sides may or may not be the same. If a polygon's sides are all the same length and its angles are all the same size, it is called a **regular polygon.** The figure to the right in each row below is a regular polygon.

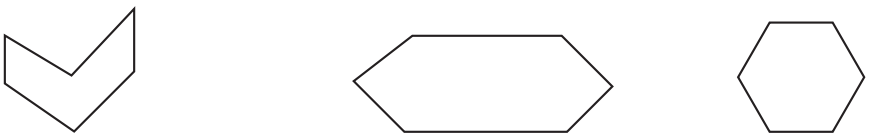


Three-sided polygons are **triangles.**

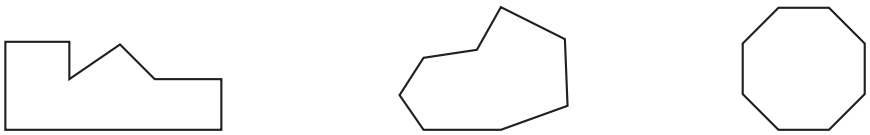
Four-sided polygons are **quadrilaterals.**



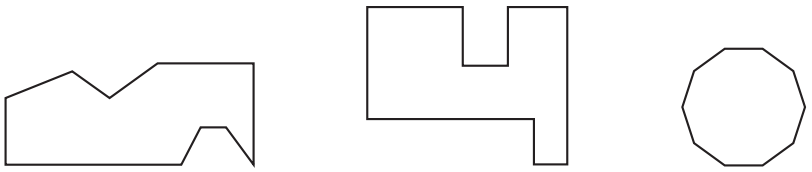
Five-sided polygons are **pentagons.**



Six-sided polygons are **hexagons.**



Eight-sided polygons are **octagons.**



Ten-sided polygons are **decagons.**

Classify Describe the angles that form a regular triangle, a rectangle, and a regular hexagon.

Example 2

What kind of a polygon is a square?

A square has four sides, so a square is a **quadrilateral.** In fact, a square is a regular quadrilateral.

Each corner of a polygon is called a *vertex* (plural: *vertices*). A polygon has as many vertices as it has sides.

Classify Describe the line segments that form a square.

Example 3

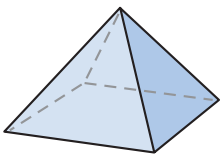
An octagon has how many more vertices than a pentagon?

An octagon has eight sides and eight vertices. A pentagon has five sides and five vertices. So an octagon has **3 more vertices** than a pentagon.

Classify Describe the line segments that form a regular pentagon and a regular hexagon.

Example 4

Name the polygons that form this pyramid.



This pyramid has 5 faces. The base is a **square.** The other four faces are **triangles.**

Analyze Describe the angles and the line segments that form this pyramid.

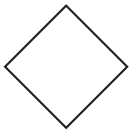
Lesson Practice

Draw an example of each of these polygons:

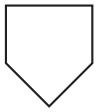
a. triangle **b.** quadrilateral **c.** pentagon

d. hexagon **e.** octagon **f.** decagon

Name each polygon shown and describe its angles:

g. 

h. 

i. 

j. 

k. Which figures in problems **g–j** appear to be regular polygons?

l. What common street sign has the shape of the polygon in problem **j?**

m. A decagon has how many more vertices than a hexagon?

Written Practice

Distributed and Integrated

1. (Inv. 2) Three feet equals 1 yard. A car that is 15 feet long is how many yards long?

***2.** (47) **Connect** Write four multiplication/division facts using the numbers 3, 10, and 30.

***3.** (35) **Analyze** Nevaeh had six quarters, three dimes, and fourteen pennies. How much money did she have in all?

4. (1, 10) What is the sum of the even numbers that are greater than 10 but less than 20?

***5.** (59) **Estimate** Round $7.15 and $5.94 to the nearest dollar, and then add.

***6.** (40) **Model** Erin opened 1 gallon of milk and began filling glasses. Each glass held 1 cup of milk. Two cups equals a pint. Two pints equals a quart. Four quarts equals a gallon. How many glasses could Erin fill? Use containers to solve.

7. (37) To what mixed number is the arrow pointing?

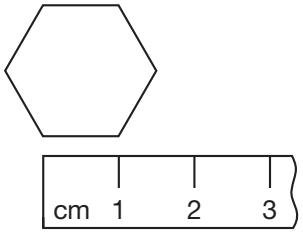
***8.** (61) The cornbread was cut into 12 equal pieces. Seven of the pieces were eaten. What fraction of the cornbread was left?

*9. The product of 4 and 3 is how much greater than the sum of 4 and 3?
(31, 38)

*10. What is the sum of 92 and $\sqrt{9}$?
(Inv. 3, 62)

*11. **a.** **Classify** What is the name of this polygon?

b. Each side is the same length. What is the perimeter of this polygon?



*12. Roger picked 56 berries in 8 minutes. At that rate, how many berries did he pick in 1 minute?
(60)

*13. Chanisse picked 11 berries in 1 minute. At that rate, how many berries could she pick in 5 minutes?
(57)

14. $\$40.00 - d = \$2.43$
(24, 52)

*15. $5 \times n = 15 + \sqrt{25}$
(Inv. 3, 61)

*16. $6 \times 4 \times 10$
(62)

*17. 5^3
(62)

18. $3.5 + 2.45$
(50)

19. $1.95 - 0.4$
(50)

20. $\$1.00 - (\$0.36 + \$0.57)$
(43, 45)

*21. 349×8
(58)

*22. $\$7.60 \times 7$
(58)

23. $6\overline{)34}$
(53)

24. $8\overline{)62}$
(53)

25. $5\overline{)24}$
(53)

26. $\frac{63}{7}$
(47)

*27. **Explain** Vans will be used to carry 22 soccer players to a game. Each van can carry 5 players. Write and solve an equation to find the least number of vans that will be needed. Then explain your answer.
(53)

*28. **Multiple Choice** Which of these numbers is a multiple of 10?
(55)

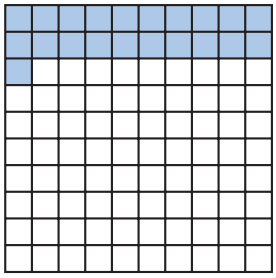
A 3 **B** 5 **C** 15 **D** 40

29. *(Inv. 4, Inv. 5)*

a. What fraction of the large square is shaded?

b. What decimal of the whole grid is not shaded?

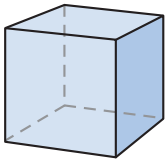
c. What percent of the large square is not shaded?



***30.** *(23, 63)*

a. **Classify** What polygons form this figure?

b. Describe the angles and line segments of this figure.



Early Finishers

Real-World Connection

Find three classroom objects that are examples of different types of polygons.

a. Name each object.

b. Draw a picture of each object. Each picture should show the number of sides each object has.

c. Label each drawing with the name of the polygon it represents.

LESSON 64

• Division with Two-Digit Answers, Part 1

Power Up

facts

Power Up J

count aloud

When we count by fives from 1, we say the numbers 1, 6, 11, 16, and so on. Count by fives from 1 to 51.

mental math

Multiply four numbers in problems **a–c.**

a. **Number Sense:** $6 \times 4 \times 10 \times 10$

b. **Number Sense:** $3 \times 4 \times 10 \times 10$

c. **Number Sense:** $4 \times 5 \times 10 \times 10$

d. **Money:** Alex had $10.00. Then he bought a cap for $6.87. How much money does Alex have left?

e. **Time:** J'Narra must finish the test by 2:30 p.m. If it is 2:13 p.m., how many minutes does she have left to finish?

f. **Measurement:** Five feet is 60 inches. How many inches tall is a person whose height is 5 feet 4 inches?

g. **Estimation:** Choose the more reasonable estimate for the width of a computer keyboard: 11 inches or 11 feet.

h. **Calculation:** $\sqrt{49} + 6 + 37 + 99$

problem solving

Choose an appropriate problem-solving strategy to solve this problem. Shamel is making lemonade for her lemonade stand. The package of powdered lemonade says that each package makes 1 quart of lemonade. If Shamel wants to make $1\frac{1}{2}$ gallons of lemonade, how many packages of powdered lemonade will she need? Explain how you found your answer.

New Concept

In this lesson we will learn a pencil-and-paper method for dividing a two-digit number by a one-digit number. We will demonstrate the method as we solve this problem:

> *The seventy-eight fifth-graders at Washington School will be divided equally among three classrooms. How many students will be in each room?*

There are three numbers in this "equal groups" problem: the total number of students, the number of classrooms, and the number of students in each classroom.

Formula:
Number of groups $\times$ Number in each group $=$ Total

Problem:
3 classrooms $\times$ n students in each classroom $=$ 78 students

Reading Math

We can write the related equation $78 \div 3 = n$ to represent this problem.

To find the number of students in each classroom, we divide 78 by 3.

$$3\overline{)78}$$

Thinking Skill

Discuss

Why do we write the first digit of the quotient in the tens place?

For the first step we ignore the 8 and divide 7 by 3. We write "2" above the 7. Then we multiply 2 by 3 and write "6" below the 7. Then we subtract and write "1."

$$\begin{array}{r} 2 \\ 3\overline{)78} \\ \underline{6} \\ 1 \end{array}$$

Next we "bring down" the 8, as shown here. Together, the 1 and 8 form 18.

$$\begin{array}{r} 2 \\ 3\overline{)78} \\ \underline{6}\downarrow \\ 18 \end{array}$$

Thinking Skill

Verify

Why do we write the second digit of the quotient in the ones place?

Now we divide 18 by 3 and get 6. We write the 6 above the 8 in 78. Then we multiply 6 by 3 and write "18" below the 18.

$$\begin{array}{r} 26 \\ 3\overline{)78} \\ \underline{6} \\ 18 \\ \underline{18} \\ 0 \end{array}$$

We subtract and find that the remainder is zero. This means that if the students are divided equally among the classrooms, there will be 26 students in each classroom.

$$78 \div 3 = 26$$

Since division facts and multiplication facts form fact families, we may arrange these three numbers to form a multiplication fact.

$$3 \times 26 = 78$$

We can multiply 3 by 26 to check our work.

$$\begin{array}{r} {\scriptstyle 1}\\ 26 \\ \times\ \ 3 \\ \hline 78 \end{array} \quad \text{check}$$

Example 1

An 87-acre field is divided into 3 equal parts. A different crop will be planted in each part. How many acres is one part of the field?

For the first step, we ignore the 7. We divide 8 by 3, multiply, and then subtract. Next we bring down the 7 to form 27. Now we divide 27 by 3, multiply, and subtract again.

$$\begin{array}{r} 29 \\ 3\overline{)87} \\ \underline{6}\downarrow \\ 27 \\ \underline{27} \\ 0 \end{array}$$

The remainder is zero, so we see that one part of the field is **29 acres.**

Now we multiply 3 by 29 to check our work. If the product is 87, we can be confident that our division was correct.

$$\begin{array}{r} {\scriptstyle 2}\\ 29 \\ \times\ \ 3 \\ \hline 87 \end{array} \quad \text{check}$$

Notice that there is no remainder when 87 is divided by 3; there is no remainder because 87 is a multiple of 3. We cannot identify the multiples of 3 by looking at the last digit because the multiples of 3 can end with any digit. However, adding the digits of a number can tell us whether a number is a multiple of 3. If the sum is a multiple of 3, then so is the number. For example, adding the digits in 87 gives us 15 (8 + 7 = 15). Since 15 is a multiple of 3, we know that 87 is a multiple of 3.

Example 2

Four students can sit in each row of seats in a school bus. Thirty-eight students are getting on the bus. If each student sits in the first available seat, what is a reasonable estimate of the number of rows of seats that will be filled?

We are asked for a reasonable estimate, so we can use compatible numbers. Since 38 and 4 are not compatible for division, we choose a number near 38 that divides evenly by 4. Nearby multiples of 4 are 36 and 40. Using one of these multiples, we find that a reasonable estimate of the number of rows that will be filled is **9** or **10.**

Example 3

Which of these numbers can be divided by 3 with no remainder?

A 56 **B** 64 **C** 45 **D** 73

We add the digits of each number:

A $5 + 6 = 11$ **B** $6 + 4 = 10$ **C** $4 + 5 = 9$ **D** $7 + 3 = 10$

Of the numbers 11, 10, and 9, only 9 is a multiple of 3. So the only choice that can be divided by 3 with no remainder is **45.**

Lesson Practice

Divide:

a. $3\overline{)51}$ **b.** $4\overline{)52}$ **c.** $5\overline{)75}$

d. $3\overline{)72}$ **e.** $4\overline{)96}$ **f.** $2\overline{)74}$

g. **Connect** Find the missing factor in this equation: $3n = 45$

h. **Multiple Choice** Which of these numbers can be divided by 3 with no remainder? How do you know?

A 75 **B** 76

C 77 **D** 79

i. Each row of desks in a classroom can seat six students. Twenty-nine students are entering the classroom. If each student sits in the first available seat, what is a reasonable estimate of the number of rows of seats that will be filled? Explain your answer.

Written Practice

Distributed and Integrated

***1.** (34) A square mile is twenty-seven million, eight hundred seventy-eight thousand, four hundred square feet. Use digits to write this number.

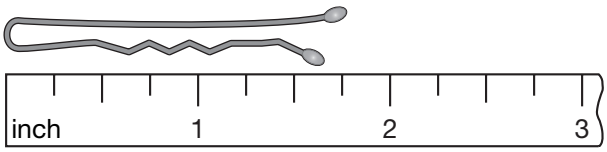
2. (49) The tree was one hundred thirteen paces away. If each pace was 3 feet, how many feet away was the tree?

3. (25, 41) Tracey's baseball-card album will hold five hundred cards. Tracey has three hundred eighty-four cards. How many more cards will fit into the album? Write an equation.

4. (52, 54) The trip lasted 21 days. How many weeks did the trip last?

***5.** (49) A stop sign has the shape of an octagon. How many sides do seven stop signs have?

***6.** (39) Find the length of this hairpin to the nearest quarter inch.



7. (16, 33) Write 406,912 in expanded form. Then use words to write the number.

***8.** (Inv. 2) One foot equals 12 inches. If each side of a square is 1 foot long, then what is the perimeter of the square in inches?

***9.** (59) **Estimate** During a school fundraiser, a group of students worked for 90 minutes and washed 8 cars. What is a reasonable estimate of the number of minutes the students spent washing each car? Explain why your answer is reasonable.

***10.** (56) **Represent** Compare: $\frac{3}{6}$ ◯ $\frac{1}{2}$. Draw and shade two congruent circles to show the comparison.

11. (33) Compare:

a. 614 ◯ 609

b. 88 ◯ 106

***12.** (11, 30) **Explain** Last week Ms. Willyard graded some papers. This week she graded 47 more papers. In these two weeks, Ms. Willyard graded 112 papers altogether. How many papers did she grade last week? Explain why your answer is reasonable.

13. (43, 51)
$$\begin{array}{r} \$32.47 \\ +\ \$67.54 \\ \hline \end{array}$$

14. (52)
$$\begin{array}{r} 51{,}036 \\ -\ 7{,}648 \\ \hline \end{array}$$

15. (50)
$$\begin{array}{r} 53.6 \\ 2.9 \\ 97.4 \\ 8.8 \\ +\ 436.1 \\ \hline \end{array}$$

***16.** (41) $5n = 75$

***17.** (64) $3\overline{)84}$

***18.** (64) $4\overline{)92}$

19. (53) $6\overline{)58}$

***20.** (58)
$$\begin{array}{r} 257 \\ \times\ \ 5 \\ \hline \end{array}$$

***21.** (58)
$$\begin{array}{r} \$7.09 \\ \times\ \ 3 \\ \hline \end{array}$$

22. (58)
$$\begin{array}{r} \$334 \\ \times\ \ 9 \\ \hline \end{array}$$

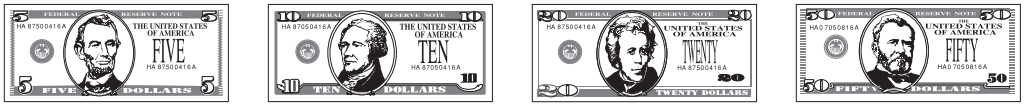
***23.** (64) $2\overline{)36}$

24. (41) $4n = 36$

***25.** (62) $4^2 + 2^3$

26. (43, 45) $3.5 - (2.4 - 1.3)$

***27.** (36) Look at these bills. List all of the different ways to pair two bills.



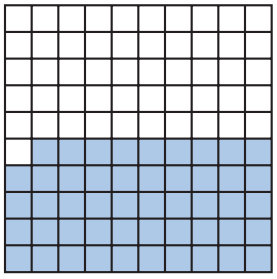
***28.** (61) Three fourths of the game was over. What fraction of the game remained?

29. (Inv. 4, Inv. 5)

a. What fraction of the large square is shaded?

b. What decimal number is represented by the shaded part of the square?

c. What percent of the large square is not shaded?



***30.** (55) **Multiple Choice** The first two prime numbers are 2 and 3. The next two prime numbers are ____.

A 4 and 5 **B** 5 and 6 **C** 5 and 7 **D** 7 and 9

LESSON 65

• Division with Two-Digit Answers, Part 2

Power Up

facts Power Up J

count aloud Count by fives from 1 to 51.

mental math Multiply two numbers ending in zero in problems **a–d.** (Example: 30×40 equals 3×10 times 4×10. We rearrange the factors to get $3 \times 4 \times 10 \times 10$, which is 1200.)

a. **Number Sense:** 40×40

b. **Number Sense:** 30×50

c. **Number Sense:** 60×70

d. **Number Sense:** 40×50

e. **Powers/Roots:** $2^2 + 2$

f. **Money:** \$6.48 + \$2.39

g. **Estimation:** Each bottled water costs 99¢. If Ms. Hathcoat buys 1 bottle for each of her 24 students, about how much money will she spend?

h. **Calculation:** $\sqrt{64} - 6 + 37 + 61$

problem solving Choose an appropriate problem-solving strategy to solve this problem. Jamisha paid a dollar for an item that cost 44¢. If she got back four coins in change, what should the four coins have been?

New Concept

The numbers in a division problem are named the **divisor,** the **dividend,** and the **quotient.**

$$\begin{array}{r} \text{quotient} \\ \text{divisor}\overline{)\,\text{dividend}} \end{array} \qquad \text{dividend} \div \text{divisor} = \text{quotient}$$

$$\frac{\text{dividend}}{\text{divisor}} = \text{quotient}$$

If we calculate how to divide 78 students among 3 classrooms, then 78 becomes the dividend and 3 becomes the divisor. The result, 26, is the quotient.

$$\begin{array}{rl} & 26 \leftarrow \text{quotient} \\ \text{divisor} \rightarrow & 3\overline{)78} \leftarrow \text{dividend} \end{array}$$

The dividend is the number being divided. The divisor is the number by which the dividend is divided. The quotient is the result of the division.

Example 1

Identify the 8 in each of these problems as the *divisor, dividend,* or *quotient:*

a. $8 \div 2 = 4$ **b.** $8\overline{)24}$ **c.** $\frac{40}{5} = 8$

a. dividend **b. divisor** **c. quotient**

Analyze Write a multiplication equation and a division equation using the words divisor, dividend, and quotient.

We solve the following problem by dividing:

On a three day bike trip Hans rode 234 kilometers. Hans rode an average of how many kilometers each day?

We find the answer by dividing 234 by 3.

$$3\overline{)234}$$

Thinking Skill

Discuss

Why do we write the first digit of the quotient in the tens place?

To perform the division, we begin by dividing $3\overline{)23}$. We write "7" above the 3 of 23. Then we multiply and subtract.

$$\begin{array}{r} 7 \\ 3\overline{)234} \\ 21 \\ \hline 2 \end{array}$$

Next we bring down the 4.

$$\begin{array}{r} 7 \\ 3\overline{)234} \\ 21\downarrow \\ \hline 24 \end{array}$$

Now we divide 24 by 3. We write "8" above the 4. Then we multiply and finish by subtracting.

Thinking Skill

Verify

Why do we write the second digit of the quotient in the ones place?

$$\begin{array}{r} 78 \\ 3\overline{)234} \\ \underline{21} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

We find that Hans rode an average of 78 kilometers each day.

We can check our work by multiplying the quotient, 78, by the divisor, 3. If the product is 234, then our division answer is correct.

$$\begin{array}{r} 78 \\ \times\ \ 3 \\ \hline 234 \end{array} \quad \text{check}$$

Example 2

On a 9-day bike trip through the Rocky Mountains, Vera and her companions rode 468 miles. They rode an average of how many miles per day?

Vera and her companions probably rode different distances each day. By dividing 468 miles by 9, we find how far they traveled if they rode the same distance each day. This is called the *average distance.* We begin by finding $9\overline{)46}$. We write "5" above the 6 in 46. Then we multiply and subtract.

$$\begin{array}{r} 5 \\ 9\overline{)468} \\ \underline{45} \\ 1 \end{array}$$

Next we bring down the 8. Now we divide 18 by 9.

$$\begin{array}{r} 52 \\ 9\overline{)468} \\ \underline{45}\downarrow \\ 18 \\ \underline{18} \\ 0 \end{array}$$

We find that they rode an average of **52 miles** per day.

We check the division by multiplying 52 by 9, and we look for 468 as the answer.

$$\begin{array}{r} {\scriptstyle 1} \\ 52 \\ \times\ \ 9 \\ \hline 468 \end{array} \quad \text{check}$$

Connect Why can we use multiplication to check a division problem?

Notice in Example 2 that there is no remainder when 468 is divided by 9. That is because 468 is a multiple of 9. Just as we identified multiples of 3 by adding the digits of a number, we can identify multiples of 9 by adding the digits of a number. For the number 468, we have

$$4 + 6 + 8 = 18$$

The sum 18 is a multiple of 9, so 468 is a multiple of 9.

Example 3

Which of these numbers is a multiple of 9?

A 123 **B** 234 **C** 345 **D** 456

We add the digits of each number:

A $1 + 2 + 3 = 6$ **B** $2 + 3 + 4 = 9$

C $3 + 4 + 5 = 12$ **D** $4 + 5 + 6 = 15$

The sums 6, 9, and 12 are all multiples of 3, but only 9 is a multiple of 9. Therefore, only **234** is a multiple of 9 and can be divided by 9 without a remainder.

Example 4

Each day, some of the students in Montrelyn's class order milk at snack time and at lunch time. Each month, a total of 192 pints of milk are delivered to Montrelyn's classroom. About how many quarts of milk are delivered each month? Explain why your answer is reasonable.

We are given a number of pints and asked for a number of quarts. Since there are 2 pints in a quart, we divide 192 pints by 2. We are not asked for an exact answer, so we can estimate. Since 192 is nearly 200, we can arrange 200 pints into groups of 2 pints by dividing. The answer is **about 100 quarts.**

Analyze About how many gallons is 192 pints? Explain your reasoning.

Lesson Practice

In the division fact $32 \div 8 = 4$,

a. what number is the divisor?

b. what number is the dividend?

c. what number is the quotient?

Divide:

d. $3\overline{)144}$ **e.** $4\overline{)144}$ **f.** $6\overline{)144}$

g. $225 \div 5$ **h.** $455 \div 7$ **i.** $200 \div 8$

j. **Multiple Choice** Which of these numbers can be divided by 9 without a remainder? How do you know?

A 288 **B** 377 **C** 466 **D** 555

k. Find the missing factor in this equation:

$$5m = 125$$

l. An oil-change business changes the motor oil in cars and trucks. On Saturday, the business sold 157 quarts of new motor oil. About how many gallons of new motor oil were sold that day? Explain your answer.

Written Practice

Distributed and Integrated

Formulate Write and solve equations for problems **1** and **2.**

1. (52) The chef uses 3 eggs for each omelette. How many omelettes can he make with two dozen eggs?

***2.** (25) Aaliyah looked at the clock and realized that her next class would begin in 27 minutes and end in 72 minutes. How many minutes long is Aaliyah's next class?

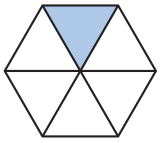
***3.** (54) Alvaro is turning three years old today. How many months old is Alvaro?

***4.** (40, 65) **Estimate** Madeline's favorite orange juice is sold in half-gallon containers. Each month, Madeline estimates that she purchases 7 containers of juice. Estimate the number of gallons of juice Madeline purchases each month. Explain your reasoning.

***5.** (60) Trudy rode her bike 36 miles in 4 hours. She rode at an average rate of how many miles per hour?

***6.** (57) **Analyze** The wagon train traveled at an average rate of 20 miles per day. At that rate, how many miles would the wagon train travel in 5 days? Make a table to solve the problem.

*7. **a.** What fraction of this hexagon is *not* shaded? (61)



b. Each side of the hexagon is 1 cm long. What is its perimeter? (Inv. 2)

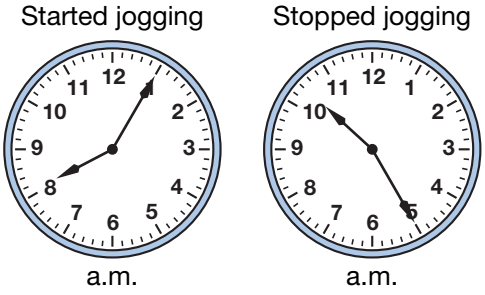
*8. **Interpret** The average amount of precipitation received each year in each of four cities is shown in the table below: (Inv. 6)

Average Annual Precipitation

City and State	Amount (to the nearest inch)
Phoenix, AZ	8
Reno, NV	7
Boise, ID	12
Albuquerque, NM	9

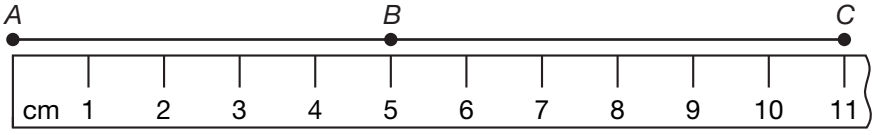
Display the data in a bar graph. Write one statement that describes the data.

*9. J'Raa started jogging early in the morning and did not stop until he returned home. How much time did J'Raa spend jogging? (27)



10. Nigel drew a circle with a radius of 18 inches. What was the diameter of the circle? (21)

11. How long is segment *BC*? (45)



*12. **Multiple Choice** Which of these words is the answer to a division problem? (65)

A product **B** dividend **C** divisor **D** quotient

***13.** (Inv. 3, 62) Compare: $27 \div 3^2 \bigcirc 27 \div \sqrt{9}$

14. (43, 51)
$$\begin{array}{r} \$97.56 \\ +\ \$\ \ 8.49 \\ \hline \end{array}$$

15. (52)
$$\begin{array}{r} \$60.00 \\ -\ \$54.78 \\ \hline \end{array}$$

16. (43)
$$\begin{array}{r} 37.64 \\ 29.45 \\ 3.01 \\ +\ 75.38 \\ \hline \end{array}$$

***17.** (65) $168 \div 3$

***18.** (65) $378 \div 7$

19. (58) 840×3

20. (58) 4×564

***21.** (58) 304×6

***22.** (65) $4\overline{)136}$

***23.** (65) $2\overline{)132}$

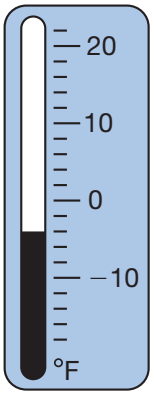
***24.** (65) $6\overline{)192}$

***25.** (61, 65) **Explain** Describe the steps for solving the equation and then solve the equation to find n.

$$7n = 50 + 34$$

***26.** (62) $12 \times 7 \times 10$

27. (18) Dimitri woke up on a cold morning and glanced out the window at the thermometer. What temperature is shown on this thermometer?



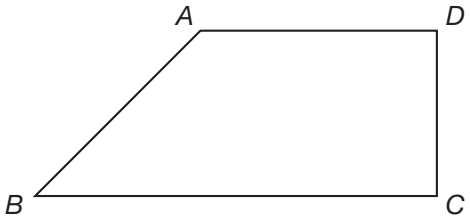
28. (36, Inv. 5)
a. Three quarters is what fraction of a dollar?

b. Three quarters is what percent of a dollar?

***29.** (63) Draw a quadrilateral. A quadrilateral has how many vertices?

***30.** (23, 45)
a. Which side of this quadrilateral is parallel to side *CB*?

b. Which angle appears to be an obtuse angle?



LESSON 66

• Similar and Congruent Figures

Power Up

facts

Power Up J

count aloud

Count down by fives from 51 to 1.

mental math

Multiply three numbers, including numbers ending in zero, in **a–c.**

a. **Number Sense:** $3 \times 10 \times 20$

b. **Number Sense:** $4 \times 20 \times 30$

c. **Number Sense:** $3 \times 40 \times 10$

d. **Powers/Roots:** $2^2 + 5^2$

e. **Geometry:** Altogether, how many sides do 3 hexagons have?

f. **Money:** Logan owes $10.00 for his club dues. He has $9.24. How much more money does Logan need?

g. **Estimation:** Lieu wants to buy 6 stickers that each cost 21¢. Lieu has $1.15. Does she have enough money to buy 6 stickers?

h. **Calculation**[1]**:** $\sqrt{16}$, $\times 2$, $\times 2$, $+ 4$, $\times 2$

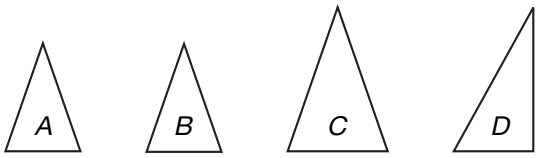
problem solving

Choose an appropriate problem-solving strategy to solve this problem. Dasha plans to use only four different colored pencils to color the states on a United States map. She has five different colored pencils from which to choose—red, orange, yellow, green, and blue. What are the combinations of four colors Dasha can choose? (There are five combinations.)

[1] As a shorthand, we will use commas to separate operations to be performed sequentially from left to right. In this case, $\sqrt{16} = 4$, then $4 \times 2 = 8$, then $8 \times 2 = 16$, then $16 + 4 = 20$, then $20 \times 2 = 40$.

New Concept

Look at these four triangles:



Figures that are the same shape are **similar.** Figures that are the same shape and the same size are *congruent.*

Triangles *A* and *B* are both similar and congruent.

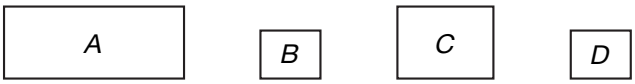
Triangles *B* and *C* are not congruent because they are not the same size. However, they are similar because they are the same shape. We could look at triangle *B* through a magnifying glass to make triangle *B* appear to be the same size as triangle *C*.

Triangle *A* and triangle *D* are not congruent and they are not similar. Neither one is an enlarged version of the other. Looking at either triangle through a magnifying glass cannot make it look like the other because their sides and angles do not match.

Example

a. Which of these rectangles are similar?

b. Which of these rectangles are congruent?



a. Rectangles *B, C,* and *D* are similar. Rectangle *A* is not similar to the other three rectangles because it is not a "magnified" version of any of the other rectangles.

b. Rectangle *B* and **rectangle *D*** are congruent because they have the same shape and size.

Activity

Determining Similarity and Congruence

Material needed:

- **Lesson Activity 29**

Model Look at the shapes on the left side of **Lesson Activity 29.** Compare each shape to the figure next to it on the right, and answer each question below.

1. Is the first shape similar to the bike sign? Is the shape congruent to the bike sign? Check your answers by cutting out the shape on the left and placing it on top of the bike sign. Describe and record the result.

2. Is the triangle similar to the yield sign? Is the triangle congruent to the yield sign? Check your answers by cutting out the triangle and placing it on top of the yield sign. Describe and record the result.

3. **Discuss** How do you know the octagon on the left is congruent to the stop sign? Are these shapes similar?

Lesson Practice

Refer to the figures below to answer problems **a** and **b.**

a. Which of these triangles appear to be similar?

b. Which of these triangles appear to be congruent?

Written Practice

Distributed and Integrated

Formulate Write and solve equations for problems **1** and **2.**

(48) **1.** Lobo works 8 hours each day and earns $18 for each hour he works. What amount of income does Lobo earn each day?

(52, 65) ***2.** Every third bead on the necklace was red. There were one hundred forty-one beads in all. How many beads were red? (Make equal groups of three.)

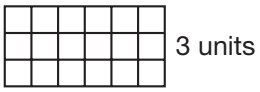
3. (Inv. 5) Twenty-five percent of this square is shaded. What percent of the square is not shaded?

***4.** (25) **Represent** In one day, Liliana drove 20 kilometers north and then 15 kilometers south. How far was Liliana from where she started? Draw a diagram to solve the problem.

5. (27) At 11:45 a.m. Dequon glanced at the clock. His doctor's appointment was in $2\frac{1}{2}$ hours. At what time was his appointment?

***6.** (Inv. 2, Inv. 3)

a. **Analyze** In the figure below, we do not state the size of the units used to measure the rectangle. Find the perimeter and area of the rectangle. Label your answers with *units* or *square units.*



b. **Represent** The rectangle has 3 rows of 6 squares, showing that 3 and 6 are factors of 18. Draw a rectangle arranged in two rows to show two other factors of 18.

***7.** (57) **Explain** The car could go 30 miles on 1 gallon of gas. How far could the car go on 8 gallons of gas? Explain your thinking.

***8.** (61) Two sevenths of the crowd cheered wildly. The rest of the crowd stood quietly. What fraction of the crowd stood quietly?

9. (3, 10) How many different three-digit numbers can you write using the digits 4, 2, and 7? Each digit may be used only once. Label the numbers you write as even or odd.

***10.** (56) **Represent** Compare: $\frac{1}{2} \bigcirc \frac{2}{5}$. Draw and shade two congruent rectangles to show the comparison.

11. (61) $n + 2 = 3 \times 12$

12. (45, 50) $6.42 - (3.3 - 1.5)$

***13.** (Inv. 3, 62) $\sqrt{81} + 82 + 3^2$

14. (43) $\$10 - 10¢$

15. (52) $43{,}016 - 5987$

***16.** (62) $24 \times 3 \times 10$

17. (58) $\$4.86 \times 7$

18. (58) 307×8

19. (58) $\$460 \times 9$

***20.** (65) $2\overline{)152}$

***21.** (65) $6\overline{)264}$

***22.** (41, 64) $4w = 56$

***23.** (65) $230 \div 5$

***24.** (64) $91 \div 7$

***25.** (65) $135 \div 3$

26. (20, 35) **a.** Write 8¢ using a dollar sign and a decimal point.

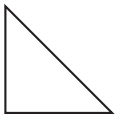
b. Round $11.89 to the nearest dollar.

***27.** (35, Inv. 4) **Represent** Use words to name each number:

a. $2\frac{3}{10}$ **b.** 2.3

***28.** (66) **a.** **Multiple Choice** Which two triangles are congruent?

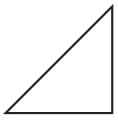
A



B

C

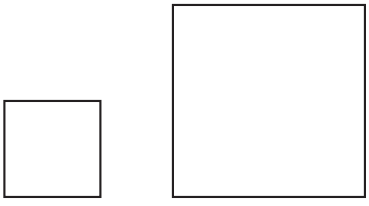
D



b. **Explain** Explain your answer to part **a.**

***29.** (63) **Represent** Draw a pentagon. A pentagon has how many vertices?

***30.** (66) **Conclude** Are all squares similar? Why or why not?



Early Finishers

Real-World Connection

Road signs often have the same shape, but they may not have the same size. Look at the road signs below. Find two signs that are congruent and two other signs that are similar but not congruent.

LESSON 67

• Multiplying by Multiples of 10

Power Up

facts Power Up I

count aloud When we count by fives from 2, we say the numbers 2, 7, 12, 17, and so on. Count by fives from 2 to 52.

mental math Multiply numbers ending in two zeros by numbers ending in one zero in **a–c.**

a. **Number Sense:** 200×10

b. **Number Sense:** 300×20

c. **Number Sense:** 400×50

d. **Percent:** 50% of \$10

e. **Percent:** 25% of \$10

f. **Percent:** 10% of \$10

g. **Estimation:** Estimate the total cost of two items priced at \$3.88 each and one item priced at \$5.98.

h. **Calculation:** 4^2, + 34, + 72, − 24

problem solving Choose an appropriate problem-solving strategy to solve this problem. Mathea exercised for 50% of an hour. For 50% of her exercise time, she was running. For how many minutes was Mathea exercising? For how many minutes was she running?

New Concept

We remember that the multiples of 10 are the numbers we say when we count by tens starting from 10. The last digit in every multiple of 10 is a zero. The first five multiples of 10 are 10, 20, 30, 40, and 50.

We may think of 20 as 2 × 10. So to find 34 × 20, we may look at the problem this way:

$$34 \times 2 \times 10$$

We multiply 34 by 2 and get 68. Then we multiply 68 by 10 and get 680.

Example 1

Write 25 × 30 as a product of 10 and two other factors. Then multiply.

Since 30 equals 3 × 10, we may write 25 × 30 as

$$\mathbf{25 \times 3 \times 10}$$

Three times 25 is 75, and 75 times 10 is **750.**

Analyze Is 25 × (3 × 10) the same as 25 × (10 × 10 × 10)? Why or why not?

To multiply a whole number or a decimal number by a multiple of 10, we may write the multiple of 10 so that the zero "hangs out" to the right. Below we use this method to find 34 × 20.

$$\begin{array}{r} 34 \\ \times\ 20 \end{array} \longleftarrow \text{zero "hangs out" to the right}$$

We first write a zero in the answer directly below the "hanging" zero.

$$\begin{array}{r} 34 \\ \times\ 20 \\ \hline 0 \end{array}$$

Then we multiply by the 2 in 20.

$$\begin{array}{r} 34 \\ \times\ 20 \\ \hline 680 \end{array}$$

Verify Is 20 the same as 10 × 10? Why or why not?

Example 2

To complete a spelling test, 30 students each wrote 34 different words. How many spelling words will the teacher check altogether?

We write the multiple of 10 as the bottom number and let the zero "hang out."

$$\begin{array}{r} 34 \\ \times\ 30 \end{array}$$

Next we write a zero in the answer directly below the zero in 30. Then we multiply by the 3. The teacher will check **1020 words.**

$$\begin{array}{r} {\scriptstyle 1} \\ 34 \\ \times\ 30 \\ \hline 1020 \end{array}$$

Justify How could you check the answer?

Example 3

A member of a school support staff ordered 20 three-ring binders for the school bookstore. If the cost of each binder was $1.43, what was the total cost of the order?

We write the multiple of 10 so that the zero "hangs out." We write a zero below the bar, and then we multiply by the 2. We place the decimal point so that there are two digits after it. Finally, we write a dollar sign in front. The cost of the order was **$28.60.**

$$\begin{array}{r} \$1.43 \\ \times \quad 20 \\ \hline \$28.60 \end{array}$$

Lesson Practice

In problems **a–f,** multiply the factors.

a. 75×10

b. 10×32

c. $10 \times 53¢$

d. $\begin{array}{r} 26 \\ \times \ 20 \\ \hline \end{array}$

e. $\begin{array}{r} \$1.64 \\ \times \quad 30 \\ \hline \end{array}$

f. $\begin{array}{r} 45 \\ \times \ 50 \\ \hline \end{array}$

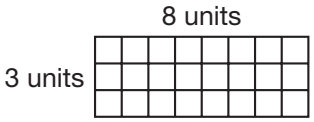
g. Write 12×30 as a product of 10 and two other factors. Then multiply.

Written Practice

Distributed and Integrated

*(52, 65) **1.** Seventy-five beans were equally divided into five pots. How many beans were in each pot?

*(Inv. 2, Inv. 3) **2. a.** **Analyze** Find the perimeter and area of this rectangle. Remember to label your answer with *units* or *square units.*



b. **Represent** Sketch a rectangle that is four units wide with the same area as the rectangle in part **a.** What is the perimeter of this new rectangle?

*(40) **3. Multiple Choice** The server placed a full pitcher of water on the table. Which of the following is a reasonable estimate of the amount of water in the pitcher?

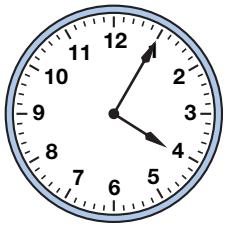
A 2 gallons **B** 2 quarts **C** 2 cups **D** 2 ounces

*4. **Multiple Choice** Which of these numbers is *not* a factor of 12?
(55)

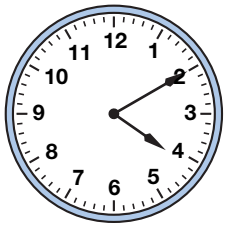
A 6 **B** 5 **C** 4 **D** 3

5. The starting time was before dawn. The stopping time was in the afternoon. What was the difference in the two times?
(27)

Starting time



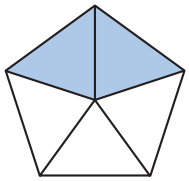
Stopping time



*6. **Represent** One square mile is 3,097,600 square yards. Use words to write that number of square yards.
(34)

7. **a.** What fraction of this pentagon is *not* shaded?
(Inv. 5, 61)

b. Is the shaded part of this pentagon more than 50% or less than 50% of the pentagon?



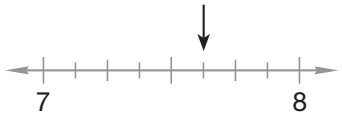
8. According to this calendar, what is the date of the last Saturday in July 2019?
(54)

JULY 2019

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

*9. **Estimate** To estimate the product of two factors, a student rounded one factor down and left the other factor unchanged. Was the estimate greater than the exact product or less than the exact product? Give an example to support your answer.
(59)

10. **Represent** To what mixed number is the arrow pointing?
(37)



*11. (59) **Justify** Sofia estimated that the exact product of 4×68 is close to 400 because 68 rounded to the nearest hundred is 100, and $4 \times 100 = 400$. Was Sofia's estimate reasonable? Explain why or why not.

*12. (Inv. 1, 62) Compare: $2^3 \bigcirc 2 \times 3$

13. (43) $\$6.25 + \$4 + \$12.78$

14. (50) $3.6 + 12.4 + 0.84$

15. (24, 52)
$$\begin{array}{r} \$30.25 \\ -\quad b \\ \hline \$13.06 \end{array}$$

16. (52)
$$\begin{array}{r} 149{,}384 \\ -\ 98{,}765 \\ \hline \end{array}$$

17. (67)
$$\begin{array}{r} 409 \\ \times\ 70 \\ \hline \end{array}$$

18. (58) $5 \times \$3.46$

19. (58) $\$0.79 \times 6$

*20. (67) $10 \times 39¢$

*21. (64) $6\overline{)90}$

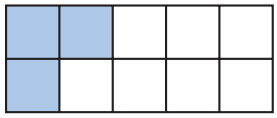
*22. (41, 64) $4w = 96$

*23. (65) $8\overline{)456}$

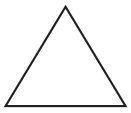
*24. (64) $95 \div 5$

*25. (65) $234 \div 3$

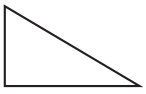
*26. (Inv. 4) Name the shaded part of this rectangle as a fraction and as a decimal.

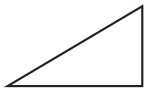


*27. (66) a. **Multiple Choice** Which two figures are congruent?

A 

B 

C 

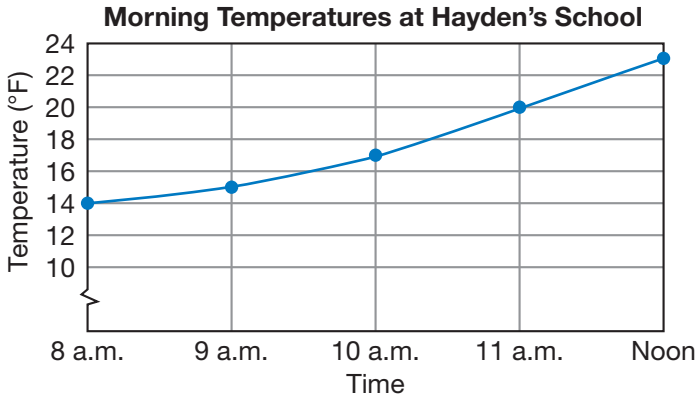
D 

b. **Conclude** Explain how you know.

28. (36) How much money is $\frac{1}{4}$ of a dollar?

*29. (63) **Represent** Draw a hexagon. A hexagon has how many vertices?

*30. (Inv. 6) **Interpret** The line graph shows the temperature at different times on a winter morning at Hayden's school. Use the graph to answer the questions that follow.



a. At what time was the first temperature of the morning recorded? What was that temperature?

b. Was the noon temperature warmer or colder than the 10 a.m. temperature? How many degrees warmer or colder was the noon temperature?

Early Finishers

Real-World Connection

Marla bought a new protein shake with ten times the amount of protein as her old protein shake.

a. If the old protein shake has 3.25 grams of protein, how many grams of protein does Marla's new shake have?

b. What equation did you use to solve?

LESSON 68

• Division with Two-Digit Answers and a Remainder

Power Up

facts Power Up I

count aloud Count down by fives from 52 to 2.

mental math

a. **Number Sense:** 10 × 20 × 30

b. **Number Sense:** 250 × 10

c. **Money:** Shatavia had $5.00. Then she spent $3.79. How much did she have left?

d. **Money:** Tan bought a scorebook for $6.48 and a whistle for $2.84. How much did he spend?

e. **Geometry:** What is the perimeter of a square with 9-inch sides? Express your answer in feet.

f. **Time:** How many years is 1 century plus 4 decades?

g. **Estimation:** Estimate 193 × 5 by rounding 193 to the nearest hundred and then multiplying.

h. **Calculation:** 18 ÷ 9, × 6, × 6

problem solving Choose an appropriate problem-solving strategy to solve this problem. Stephanie solved an addition problem and then erased some of the digits from the problem. She gave it to Ian as a problem-solving exercise. Copy Stephanie's problem on your paper, and find the missing digits for Ian.

```
  7_6
+ _4_
 _45
```

New Concept

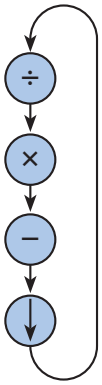
The pencil-and-paper method we use for dividing has four steps: divide, multiply, subtract, and bring down. These steps are repeated until the division is complete.

Step 1: Divide.

Step 2: Multiply.

Step 3: Subtract.

Step 4: Bring down.



For each step we write a number. When we finish Step 4, we go back to Step 1 and repeat the steps until there are no more digits to bring down. The number left after the last subtraction is the remainder. We show the remainder in the division answer by writing it with an uppercase "R" in front.

Example 1

Divide: $5\overline{)137}$

Step 1: Divide 13 by 5 and write "2."

Step 2: Multiply 2 by 5 and write "10."

Step 3: Subtract 10 from 13 and write "3."

Step 4: Bring down 7 to make 37.

$$\begin{array}{r} 2 \\ 5\overline{)137} \\ 10\downarrow \\ \hline 37 \end{array}$$

Now we repeat the same four steps:

Step 1: Divide 37 by 5 and write "7."

Step 2: Multiply 7 by 5 and write "35."

Step 3: Subtract 35 from 37 and write "2."

Step 4: There are no more digits to bring down, so we will not repeat the steps. The remainder is 2. Our answer is **27 R 2.**

$$\begin{array}{r} 27 \\ 5\overline{)137} \\ 10 \\ \hline 37 \\ 35 \\ \hline 2 \end{array}$$

Thinking Skill

Verify

Why do we write the first digit of the quotient in the tens place?

If we divide 137 into 5 equal groups, there will be 27 in each group. There will also be 2 extra.

To check a division answer that has a remainder, we multiply the quotient (without the remainder) by the divisor and then add the remainder. For this example, we multiply 27 by 5 and then add 2.

$$\begin{array}{r} 27 \\ \times\ 5 \\ \hline 135 \end{array} \qquad \begin{array}{r} 135 \\ +\ \ 2 \\ \hline 137 \end{array} \quad \text{check}$$

Example 2

Three hundred seventy-five fans chartered eight buses to travel to a playoff basketball game. About how many fans were on each bus if the group was divided as evenly as possible among the eight buses?

To find "about how many people," we can estimate with compatible numbers. Instead of dividing 375 by 8, we will divide 400 by 8.

$$400 \div 8 = 50$$

There will be **about 50 people** on each bus.

Lesson Practice

Divide:

a. $3\overline{)134}$ **b.** $7\overline{)240}$ **c.** $5\overline{)88}$

d. $259 \div 8$ **e.** $95 \div 4$ **f.** $325 \div 6$

g. Shou divided 235 by 4 and got 58 R 3 for her answer. Describe how to check Shou's calculation.

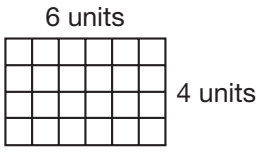
h. A wildlife biologist estimates that 175 birds live in the 9-acre marsh. What is a reasonable estimate of the number of birds in each acre of the marsh? Explain why your estimate is reasonable.

Written Practice

Distributed and Integrated

***1.** (57) **Analyze** Alphonso ran 6 miles per hour. At that rate, how far could he run in 3 hours? Make a table to solve this problem.

***2.** (Inv. 2, Inv. 3) Find the perimeter and area of this rectangle:



3. (Inv. 4) **Represent** Aletta ran 100 meters in twelve and fourteen hundredths seconds. Use digits to write her time.

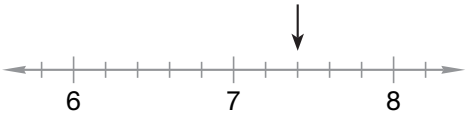
***4.** (63) Taydren drew an octagon and a pentagon. How many sides did the two polygons have altogether?

***5.** (67) 47×30 ***6.** (67) 60×39 ***7.** (67) 85×40

*8. (Inv. 5, 61) **a.** Maura ran $\frac{3}{5}$ of the course but walked the rest of the way. What fraction of the course did she walk?

b. Did Maura run more than 50% of the course or less than 50% of the course?

9. (37) **Represent** To what mixed number is the arrow pointing?



*10. (37) **Model** Draw a number line and show the locations of 0, 1, 2, $1\frac{2}{3}$, and $2\frac{1}{3}$.

11. (33) **Represent** Mount Rainier stands four thousand, three hundred ninety-two meters above sea level. Use digits to write that number.

*12. (60) Mo'Nique could make 35 knots in 7 minutes. How many knots could she make in 1 minute?

13. (59) Estimate the sum of 6810 and 9030 by rounding each number to the nearest thousand before adding.

*14. (43, 59) Estimate the sum of $12.15 and $5.95. Then find the exact sum.

15. (43, 45) $20 − ($8.95 + 75¢)

16. (43) 23.64 − 5.45

17. (48) $\begin{array}{r} 43¢ \\ \times\ \ 8 \\ \hline \end{array}$

18. (58) $\begin{array}{r} \$3.05 \\ \times\ \ \ \ 5 \\ \hline \end{array}$

19. (58) $\begin{array}{r} \$2.63 \\ \times\ \ \ \ 7 \\ \hline \end{array}$

20. (27) **Connect** Rewrite this addition problem as a multiplication problem and find the answer:

$$64 + 64 + 64 + 64 + 64$$

*21. (68) $5\overline{)96}$

*22. (68) $7\overline{)156}$

*23. (65) $3\overline{)246}$

*24. (65) $\frac{216}{6}$

*25. (41, 65) $4r = 156$

*26. (68) $195 \div 8$

*27. (39, 45) **Model** Use an inch ruler to find the lengths of segments *AB, BC,* and *AC.*

*28. (63, 66) a. **Multiple Choice** Which word makes the following sentence untrue?

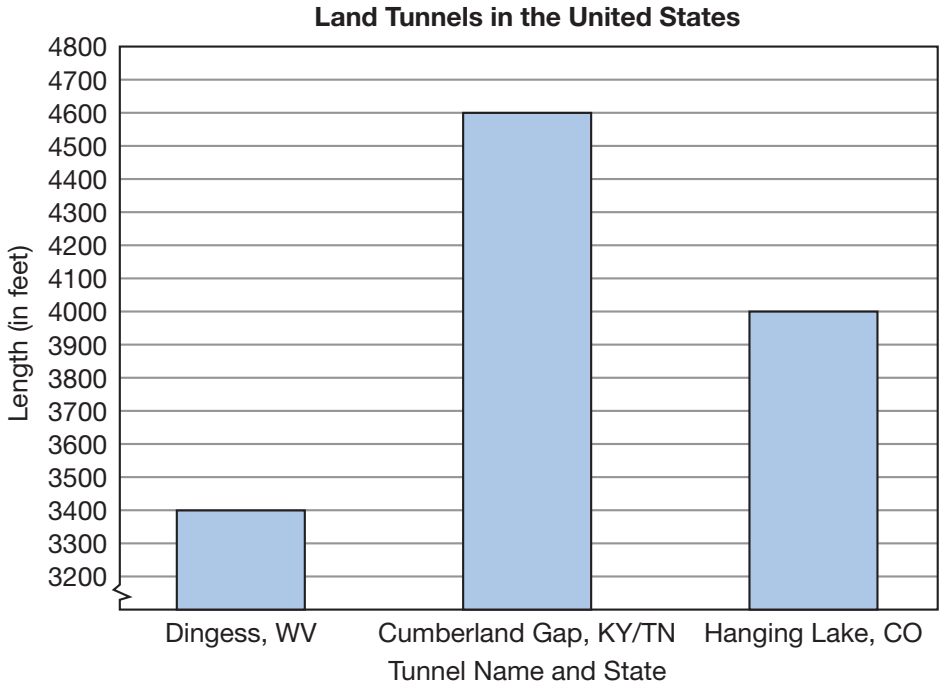
All squares are ____.

A polygons **B** rectangles **C** similar **D** congruent

b. **Explain** Explain your choice.

29. (40) Compare: 2 quarts ◯ $\frac{1}{2}$ gallon

*30. (Inv. 6) **Interpret** The lengths of three land tunnels in the United States are shown in the graph. Use the graph to answer parts **a–c.**



a. Write the names of the tunnels in order from shortest to longest.

b. How many feet longer is the Hanging Lake Tunnel than the Dingess Tunnel?

c. One mile is equal to 5280 feet. Are the combined lengths of the tunnels more than or less than 2 miles?

LESSON 69

• Millimeters

Power Up

facts Power Up I

count aloud Count down by threes from 60 to 3.

mental math

a. **Number Sense:** $12 \times 2 \times 10$

b. **Number Sense:** $20 \times 20 \times 20$

c. **Number Sense:** $56 + 9 + 120$

d. **Fractional Parts:** What is $\frac{1}{2}$ of \$60?

e. **Measurement:** Six feet is 72 inches. How many inches tall is a person whose height is 5 feet 11 inches?

f. **Measurement:** The airplane is 5500 feet above the ground. Is that height greater than or less than 1 mile?

g. **Estimation:** Xavier can read about 30 pages in one hour. If Kevin must read 58 pages, about how long will it take him? (Round your answer to the nearest hour.)

h. **Calculation:** 6^2, $-$ 18, $\div$ 9, $\times$ 50

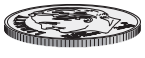
problem solving Choose an appropriate problem-solving strategy to solve this problem. The parking lot charged \$1.50 for the first hour and 75¢ for each additional hour. Harold parked the car in the lot from 11:00 a.m. to 3 p.m. How much money did he have to pay? Explain how you found your answer.

New Concept

This line segment is one centimeter long:

———

If we divide a centimeter into ten equal lengths, each equal length is **1 millimeter** long. A dime is about 1 millimeter thick.

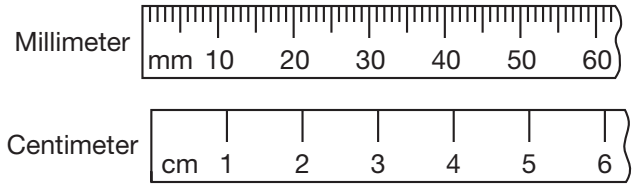
 ← 1 millimeter thick

The words *centimeter* and *millimeter* are based on Latin words. *Centum* is the Latin word for "hundred." A centimeter is one hundredth ($\frac{1}{100}$) of a meter, just as a cent is one hundredth of a dollar. *Mille* is the Latin word for "thousand." A millimeter is one thousandth ($\frac{1}{1000}$) of a meter, just as a milliliter is one thousandth of a liter.

Math Symbols

The abbreviation for centimeter is cm. The abbreviation for millimeter is mm.

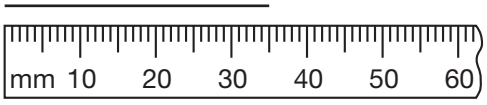
Here we show a millimeter scale and a centimeter scale:



We can see from the scales that each centimeter equals ten millimeters.

Example 1

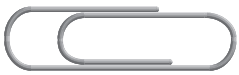
The segment below is how many millimeters long?



The length of the segment is **35 mm.**

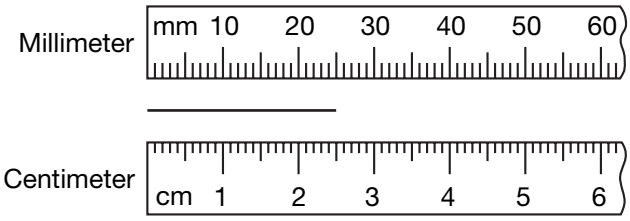
Example 2

This paper clip is 3 cm long. How many millimeters long is it?



Each centimeter is 10 mm. We multiply 10 mm by 3 to find that the length of the paper clip is **30 mm.**

Using the scales below, we see that a segment that is 25 mm long is $2\frac{5}{10}$ cm long.



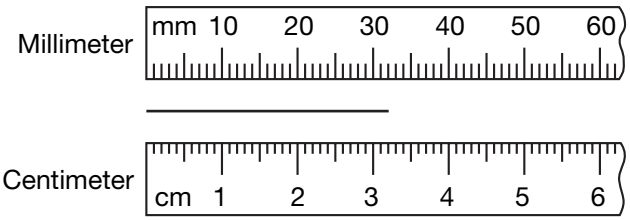
We usually write metric measures as decimal numbers instead of fractions. So a 25-mm segment is 2.5 cm long.

Example 3

Write the length of this segment

a. in millimeters.

b. in centimeters.



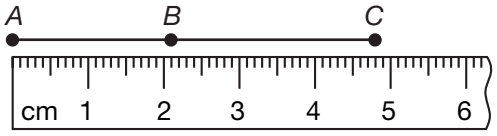
a. 32 mm

b. 3.2 cm

Connect Write 3.2 cm as a mixed number.

Example 4

Write a decimal subtraction problem that shows how to find the length of segment *BC*.



Segment *AC* is 4.8 cm long. Segment *AB* is 2.1 cm long. If we "take away" segment *AB* from segment *AC,* segment *BC* is left. We subtract 2.1 cm from 4.8 cm to find the length of segment *BC.*

$$4.8 - 2.1 = 2.7$$

We find that segment *BC* is 2.7 cm long.

Connect Write 2.7 cm as a mixed number.

Activity

Measuring with Metric Units

Model Use a ruler to estimate the length of real-world items.

1. **Estimate** Find an item in your classroom that is about 280 mm long. How many centimeters is 280 mm?

2. **Estimate** Find an item in your classroom that is about 170 mm long. How many centimeters is 170 mm?

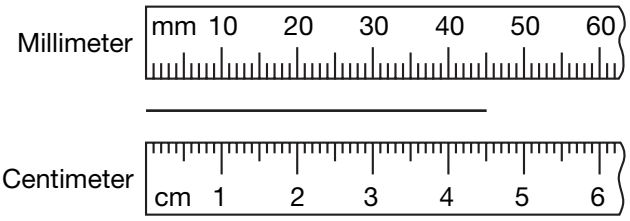
3. Find two other items in your classroom and estimate each length in millimeters and centimeters. Record your estimates. Then measure the items to see how close your estimate is to the actual measurement.

Discuss When measuring the length of an object in millimeters and in centimeters, which number is greater, the number of millimeters or the number of centimeters? Explain why.

Lesson Practice

a. The thickness of a dime is about 1 mm. Estimate the number of dimes it would take to form a stack that is about 1 cm high.

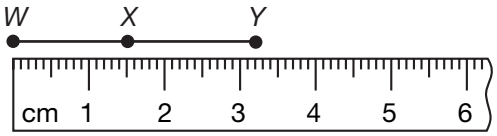
b. Write the length of this segment twice, once in millimeters and once in centimeters.



c. Each side of this square is 1 cm long. What is the perimeter of this square in millimeters?

d. The diameter of a penny is about 19 mm. How many centimeters is that?

e. Write a decimal subtraction equation that shows how to find the length of segment *XY*.



f. Write 3.4 cm as a fraction.

Written Practice

Distributed and Integrated

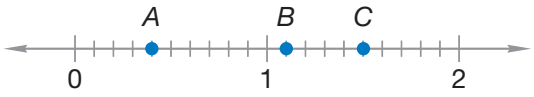
Formulate Write and solve equations for problems **1** and **2.**

(25, 30) ***1.** Celeste has three hundred eighty-four baseball cards. Will has two hundred sixty baseball cards. Celeste has how many more cards than Will?

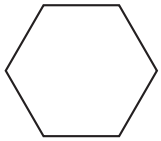
(49, 67) ***2.** Forty-two students could ride in one bus. There were 30 buses. How many students could ride in all the buses?

(69) ***3.** Kya's house key is 5.2 cm long. How many millimeters long is her house key?

*4. (Inv. 1, 37) **Represent** Write a decimal and a fraction (or a mixed number) to represent each point.

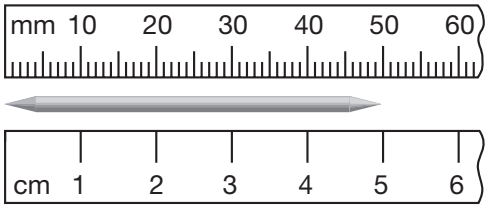


*5. (26) **Represent** Copy this hexagon and shade one sixth of it.



*6. (69) **a.** This toothpick is how many centimeters long?

b. This toothpick is how many millimeters long?



7. (Inv. 5) Twenty-five percent of the students in a class completed the science project on Thursday. All of the other students in the class completed the project on Friday. What percent of the students completed the project on Friday?

8. (Inv. 2, 49) **Analyze** One yard equals 3 feet. If each side of a square is 1 yard long, then what is the perimeter of the square in feet?

*9. (59) **Explain** The number of students enrolled at each of three elementary schools is shown in the table below.

Elementary School Enrollment

School	Number of Students
Van Buren	412
Carter	495
Eisenhower	379

Use rounding to make a reasonable estimate of the total number of students enrolled at the three schools. Explain your answer.

*10. (45, 69) Segment *AB* is 3.5 cm long. Segment *AC* is 11.6 cm long. How long is segment *BC*? Write a decimal subtraction equation and find the answer.

11. (57, 60) **a.** Hugo rode 125 miles in 5 hours. His average speed was how many miles per hour?

b. Levi could ride 21 miles in 1 hour. At that rate, how many miles could Levi ride in 7 hours?

***12.** (55) The first three prime numbers are 2, 3, and 5. What are the next three prime numbers?

13. (20, 59) **Estimate** Claudio's meal cost $7.95. Timo's meal cost $8.95. Estimate the total price for both meals by rounding each amount to the nearest dollar before adding.

***14.** (68) $250 \div 6$

***15.** (68) $100 \div 9$

16. (43)
$$\begin{array}{r} 36.2 \\ 4.7 \\ 15.9 \\ 148.4 \\ 30.5 \\ +\ \ 6.0 \\ \hline \end{array}$$

***17.** (65) $\frac{256}{8}$

***18.** (41, 64) $4w = 60$

19. (58) $9 \times \$4.63$

***20.** (67) $80 \times 29¢$

21. (52)
$$\begin{array}{r} \$10.00 \\ -\ \$\ \ 1.73 \\ \hline \end{array}$$

22. (52)
$$\begin{array}{r} 36{,}428 \\ -\ 27{,}338 \\ \hline \end{array}$$

***23.** (67)
$$\begin{array}{r} 78 \\ \times\ \ 60 \\ \hline \end{array}$$

***24.** (65) $4\overline{)328}$

***25.** (68) $7\overline{)375}$

***26.** (65) $5\overline{)320}$

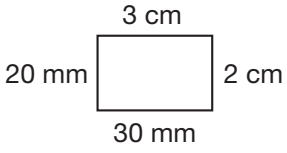
27. (61) $a + 5 = 25 + 25$

***28.** (43, 45) **Explain** Solve the equation below and describe the steps in the order you completed them.

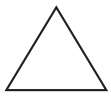
$$4.7 - (3.6 - 1.7)$$

***29.** (Inv. 2, Inv. 3) **a.** Find the perimeter of this rectangle in millimeters.

b. Find the area of this rectangle in square centimeters.



***30.** (23) **Multiple Choice** Each angle of this triangle is ____.



A acute **B** right

C obtuse **D** straight

LESSON 70

• Word Problems About a Fraction of a Group

Power Up

facts Power Up I

count aloud Count by fives from 1 to 51.

mental math

a. **Number Sense:** $21 \times 2 \times 10$

b. **Number Sense:** $25 \times 2 \times 10$

c. **Number Sense:** $12 \times 4 \times 10$

d. **Money:** $5.36 + $1.98

e. **Measurement:** Ten feet is how many inches?

f. **Estimation:** Round the prices $2.58 and $6.54 to the nearest dollar and then add to estimate the total.

g. **Estimation:** Round the prices $2.58 and $6.54 to the nearest 25 cents and then add to estimate the total.

h. **Calculation:** $9^2 + 125 + 37$

problem solving Choose an appropriate problem-solving strategy to solve this problem. Tazara has ten coins that total one dollar, but only one of the coins is a dime. What are the other nine coins? (There are two possibilities.)

New Concept

Reading Math

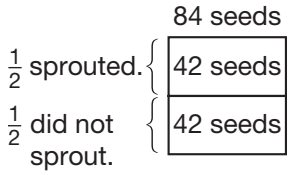
We can use fractions to name part of a whole, part of a group or number, and part of a distance.

We know that the fraction $\frac{1}{2}$ means that a whole has been divided into 2 parts. To find the number in $\frac{1}{2}$ of a group, we divide the total number in the group by 2. To find the number in $\frac{1}{3}$ of a group, we divide the total number in the group by 3. To find the number in $\frac{1}{4}$ of a group, we divide the total number in the group by 4, and so on.

Example 1

One half of the carrot seeds sprouted. If 84 seeds were planted, how many seeds sprouted?

We will begin by drawing a picture. The large rectangle stands for all the seeds. We are told that $\frac{1}{2}$ of the seeds sprouted, so we divide the large rectangle into 2 equal parts (into halves). Then we divide 84 by 2 and find that **42 seeds** sprouted.



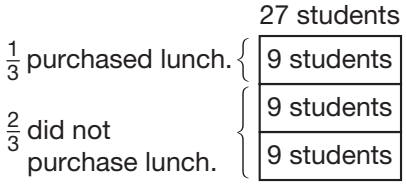
$$2\overline{)84\text{ seeds}}\quad 42\text{ seeds}$$

Discuss How can we use addition to check the answer?

Example 2

On Friday, one third of the 27 students purchased lunch in the school cafeteria. How many students purchased lunch on Friday?

We start with a picture. The whole rectangle stands for all the students. Since $\frac{1}{3}$ of the students purchased lunch, we divide the rectangle into 3 equal parts. To find how many students are in each part, we divide 27 by 3 and find that **9 students** purchased a lunch on Friday.



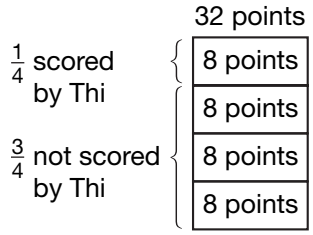
$$3\overline{)27\text{ students}}\quad 9\text{ students}$$

Justify Explain why the answer is correct.

Example 3

One fourth of the team's 32 points were scored by Thi. How many points did Thi score?

We draw a rectangle. The whole rectangle stands for all 32 points. Thi scored $\frac{1}{4}$ of the points, so we divide the rectangle into 4 equal parts. We divide 32 by 4 and find that each part is 8 points. Thi scored **8 points.**



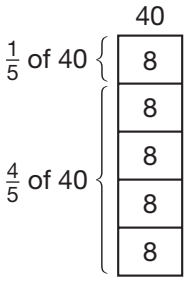
$$4\overline{)32 \text{ points}}$$
8 points

Justify Explain why the answer is correct.

Example 4

What is $\frac{1}{5}$ of 40?

We draw a rectangle to represent 40. We divide the rectangle into five equal parts, and we divide 40 by 5. Each part is 8, so $\frac{1}{5}$ of 40 is **8.**



$$5\overline{)40}$$
8

Lesson Practice

Draw a picture to solve each problem:

a. What is $\frac{1}{3}$ of 60?

b. What is $\frac{1}{2}$ of 60?

c. What is $\frac{1}{4}$ of 60?

d. What is $\frac{1}{5}$ of 60?

e. One half of the 32 children were boys. How many boys were there?

f. One third of the 24 coins were quarters. How many quarters were there?

Written Practice

Distributed and Integrated

Formulate Write and solve equations for problems **1** and **2.**

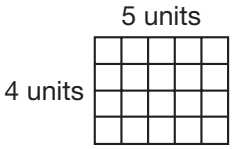
***1.** (31) There were 150 seats in the cafeteria. If 128 seats were filled, how many seats were empty?

***2.** (Inv. 4, 43) **Analyze** Anaya ran 100 meters in 12.14 seconds. Marion ran 100 meters in 11.98 seconds. Marion ran 100 meters how many seconds faster than Anaya?

3. Forty-two million is how much greater than twenty-four million?
(31, 34)

4. Keenan bought his lunch Monday through Friday. If each lunch cost $1.25, how much did he spend on lunch for the week?
(49)

***5.** Find the perimeter and area of this rectangle:
(Inv. 2, Inv. 3)



***6.** **Explain** Re'Bekka read 30 pages a day on Monday, Tuesday, and Wednesday. She read 45 pages on Thursday and 26 pages on Friday. How many pages did she read in all? Explain why your answer is reasonable.
(22)

***7.** **a.** **Represent** One half of the cabbage seeds sprouted. If 74 seeds were planted, how many sprouted? Draw a picture to solve the problem.
(Inv. 5, 70)

b. What percent of the seeds sprouted?

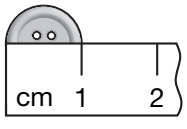
8. **Represent** Show all of the different ways these bills can be arranged in a row.
(36)

***9.** **Represent** What is $\frac{1}{6}$ of 60? Draw a picture to solve the problem.
(70)

***10.** **Analyze** Driving at a highway speed limit of 65 miles per hour, how far can a truck travel in 3 hours? Make a table to solve this problem.
(57)

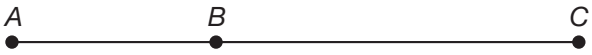
***11.** **Formulate** If a truck traveled 248 miles in 4 hours, then the truck traveled an average of how many miles each hour? Write an equation to solve this problem.
(60, 65)

***12.** **a.** What is the diameter of this shirt button in centimeters?
(69)



b. What is the radius of this shirt button in millimeters?

*13. (45, 69) Segment *AB* is 2.7 cm long. Segment *BC* is 4.8 cm long. How long is segment *AC*? Write a decimal addition equation and find the answer.



14. (43) $\$8 + \$9.48 + 79¢$

15. (50) $5.36 + 2.1 + 0.43$

16. (52)
$$\begin{array}{r} \$100.00 \\ -\ \$\ 59.47 \\ \hline \end{array}$$

17. (52)
$$\begin{array}{r} 37{,}102 \\ -\ 18{,}590 \\ \hline \end{array}$$

18. (Inv. 3, 62) $\sqrt{49} \times 2^3$

*19. (67) $\$1.63 \times 40$

*20. (67) 60×39

21. (58) $7 \times \$2.56$

*22. (68) $3\overline{)89}$

*23. (65) $9\overline{)234}$

24. (64) $\frac{90}{6}$

*25. (68) $243 \div 7$

*26. (41, 65) $5m = 355$

27. (2) $7 + n = 28$

28. (35, Inv. 4) **Represent** Write twelve and three tenths as a mixed number and as a decimal number.

*29. (55) **Multiple Choice** Which of these numbers is a factor of both 12 and 20?

A 3 **B** 4 **C** 5 **D** 6

*30. (23) **Represent** Draw a triangle that has one right angle.

Early Finishers

Real-World Connection

Leroy's class took a field trip to the aquarium. A total of 35 students and adults went on the trip. Five sevenths of the group were students.

a. How many students went on the field trip?

b. Draw a diagram to show that your answer is reasonable.

Focus on

• Collecting Data with Surveys

In Investigation 6, a pictograph displayed the favorite lunches of the students in Room 12. The information in the graph was gathered by asking students to name their favorite lunch from the school menu. The students who answered the question were participating in a **survey.** A survey is an effort to gather specific information about a group, or a **population.** People who make surveys collect information about part of the population. This part is called a **sample.** Then they draw conclusions about how the results of the survey apply to the whole population. In the favorite-lunch survey, the students in Room 12 were the sample, while all Thompson School students were the population.

Visit www.SaxonMath.com/Int4Activities for an online activity.

In this investigation you will conduct a survey of students in your class. You will need to write questions for the survey, ask the questions fairly, record the answers, and display the results of the survey. From the survey you may be able to draw conclusions about a larger population.

The way survey questions are asked can affect the results of a survey. Here are two survey questions. Describe how the answers to these questions might be different.

Which of these school lunches is your favorite?	Which lunch from the school menu is your favorite?
○ tuna ○ turkey ○ pizza ○ chicken	_____

Notice that one of the questions is a multiple-choice question. The answer is limited to the choices that are provided. The other question is open to many answers.

1. **Formulate** Write two questions that you could ask to determine students' favorite drink to have with lunch. For one question, provide options to choose from. For the other, leave the question open (do not list options). You may use the favorite-lunch questions as models.

Survey questions should be phrased without **bias;** that is, without favoring one choice over another.

2. Describe the bias in the following question:

Which drink do you prefer with lunch: cool, sweet lemonade or milk that has been out of the refrigerator for an hour?

3. **Formulate** Rewrite the question in problem **2** so that the bias is removed.

When we use a sample to find information about a larger population, we need to make sure that the sample is very similar to the population. For example, if we wanted to know the favorite TV show of kindergarten students, we would not survey a group of fourth grade students.

4. a. **Multiple Choice** For your survey, you will collect answers from students in your class. This means that your class is the sample. Which of these larger populations will probably be best represented by your survey results?

A all the students in the school

B all the school children your age in your community

C all the children your age in the country

D all the parents of the students in the class

b. **Explain** For each choice, explain why each population could or could not be represented by your sample.

When we ask our survey questions, we need to have a way to record the answers. One way to keep track of answers is with a tally sheet. On a tally sheet we make **tally marks.** A tally mark is a short vertical mark that counts as one. Two marks count as two. Four marks with a fifth, diagonal mark crossing them counts as five. Here is an example of a tally sheet for the favorite-lunch question:

Question: Which of these school lunches is your favorite?
tuna, turkey, pizza, chicken

Answer:		
	tuna	𝍸 ||||
	turkey	𝍸
	pizza	𝍸 𝍸
	chicken	𝍸 |

The question was written on the tally sheet so that it could be read to the person being interviewed. By reading the question from a sheet, we make sure that we ask the question the same way each time.

5. **Interpret** Each time a person answers the question, a tally mark is placed by the answer. Look at the tally marks for tuna. According to the tally marks, how many students named tuna as their favorite lunch?

6. **Represent** Create a tally sheet similar to the one above to show favorite drinks to have with lunch. Write a question with choices. Then list the possible answers, leaving room on the paper to tally the answers. One of the options may be "no opinion."

While gathering or combining data, we need to avoid duplicating or omitting data. Duplicating data occurs when information is counted more than once or repeated. Omitting data occurs when necessary information is left out.

7. **Analyze** Brad found that 11 students in his class had dogs for pets. Dena found that 9 students in the same class had cats for pets. Brad and Dena concluded that 20 students in the class had dogs or cats for pets. Do you think their conclusion is correct? Why or why not?

8. **Analyze** Brad and Dena also concluded that 20 students in the class have pets. Do you think this conclusion is correct? Why or why not?

Activity

Class Survey

Represent With your group, think of a survey question to ask other students.[1] Your question should not have bias. Provide at least two answer options from which the other students may choose. Make a tally sheet that contains your question and answer choices. Ask the other students your survey question. Be sure to tally the other students' answers on your tally sheet. When you have finished your survey, choose and make the appropriate graph to display the results of your survey.

Investigate Further

a. Estimate and record the temperature of a cup of water with ice and a cup of water without ice. Provide both temperatures in Celsius and Fahrenheit. Then use a thermometer to measure the actual temperatures of both cups of water. Was your original estimate correct? Compare your estimate to the actual temperatures in both Celsius and Fahrenheit.

[1] Sample topics for surveys:
- favorite sport or sports team
- favorite television show
- favorite school subject
- number of siblings in family
- how students get to school
- favorite season of the year

b. Before you start this activity, look at the clock and find your starting time. Use paper to write each multiple of 4 from 4 to 100. After you are finished, write your ending time. How much time did it take you to complete the activity? Repeat the activity by writing the multiples of 2 from 2 to 100. Which activity took longer? How much longer did it take?

c. Time yourself saying the alphabet. Look at the clock. Write down the beginning time. Then say the alphabet twice. Write down the ending time. Find the time that elapsed while you said the alphabet two times.